Charged particle multiplicity distributions in pp, $\pi^{\pm}$p and $K^{\pm}$p scattering and the approximately normal distribution

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Charged particle multiplicity distributions in pp, $\pi^{\pm}$p and $K^{\pm}$p scattering and the approximately normal distribution

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Abstract. It is shown that the approximately normal distribution for the production of j negatively charged particles describes the charged particle multiplicity data in $\pi^{\pm}$p, $K^{\pm}$p and pp scattering with laboratory momenta in the range 5–300 GeV/c. Energy dependent parametrizations of the pp and π^-p data are shown to be successful, and to become independent of the initial state as energy becomes large. It is also demonstrated that, if

$$Q(j) = \ln[\sigma(j)/\sigma(j-1)]$$

is plotted as a function of available energy, universal curves result. These are used to predict K^-p multiplicity distributions at a laboratory momentum of 150 GeV/c. A brief discussion of the importance of diffractive scattering is included.

1. Introduction

There is now a considerable body of data on charged particle multiplicity distributions in pp and π^-p scattering, as well as some experimental results for $K^{\pm}$p and π^+p scattering, when the laboratory momentum P_{LAB} lies between 5 and 300 GeV/c†.

In this paper we shall show that the simple model developed by Kaiser (1972, 1973) which makes use of the approximately normal distribution (see equation (2)), describes the data from a variety of initial states very well, when parameters are determined separately at each value of the energy.

We shall show that a more sophisticated energy dependent parametrization, as described by equations (4) and (5), reproduces all of the pp data and all of the π^-p data, provided that a diffractive mechanism is introduced that contributes an important part to the elastic amplitude and a negligible part to the production amplitudes (see figures 2 and 3). We show that the pp and π^-p parametrizations become the same as $s \to \infty$.

† In pp interactions we have used data with the following values of P_{LAB}: 5·5 GeV/c (Alexander *et al* 1967), 6·5 GeV/c (Gellert 1972), 10 GeV/c (Almeida *et al* 1968), 12·9, 18·0, 21·0, 24·1 and 28·5 GeV/c (Smith *et al* 1969), 19 GeV/c (Scandinavian collaboration 1971), 24 GeV/c (Nilssón *et al* 1966), 50 and 70 GeV/c (Soviet–French collaboration 1972), 102 GeV/c (Chapman *et al* 1972), 205 GeV/c (Charlton *et al* 1972), and 303 GeV/c (Dao *et al* 1972): for π^-p, 10 GeV/c (Bartke 1966), 16 GeV/c (ABBCCHW collaboration 1969), 20 GeV/c (Balea *et al* 1969), 25 GeV/c (Elbert *et al* 1970), 40 GeV/c (BBDHSTTUBW collaboration 1973), 50 GeV/c (France USSR/CERN–USSR collaboration 1973), and 205 GeV/c (Bogert *et al* 1973): for π^+p, 7 GeV/c (Stone *et al* 1971), 8 GeV/c (ABC collaboration 1968) and 16 GeV/c (Ballam *et al* 1971, Bracci *et al* 1972): for K^-p, 8·25 GeV/c (Fry *et al* 1972) and 33·8 GeV/c (France–USSR/CERN–USSR collaboration 1973): and for K^+p, 12·5 GeV/c (Stone *et al* 1971). All of these data, with the exception of those obtained from the 40 GeV/c π^-p propane target experiment (BBDHSTTUBW collaboration 1973), have been accumulated using liquid hydrogen bubble chambers.

Towards the end of the paper we compare the multiplicity distributions from the various initial states as a function of available energy, with a view to determining whether the model ought to be further modified to take account of diffractive production processes. We conclude that modifications of the approximately normal distribution would only be important in $\sigma(j = 0)$ and $\sigma(j = 1)$ (see later for notation), should such a study be attempted.

We note that plots of $Q(j) = \ln[\sigma(j)/\sigma(j-1)]$ as a function of available energy provide remarkable universal fits (see figure 4) and use this observation to predict the K^-p multiplicity distributions at a laboratory momentum of 150 GeV/c.

1.1. Notation

j is the number of negatively charged particles produced in a hadron–hadron collision. In π^-p and K^-p scattering, the charge exchange ($j = -1$) state is ignored. $\sigma(j)$ and $P(j)$ are the corresponding cross sections and probabilities that j negative particles will be produced. $\sqrt{s}$ is the total energy. m_b and m_t are the masses of the beam and target particles respectively. $Q = E_{av} = \sqrt{s} - m_b - m_t$ is the available energy. σ(el), σ(inel) and σ(tot) are the elastic, inelastic and total cross sections respectively.

2. The approximately normal distribution and pp scattering

Kaiser (1972, 1973) argues that, as $s \to \infty$, the probability $P(j)$ for producing j negative particles during a hadron–hadron collision, will be described by the normal distribution.

$$P(j) = \exp[-(j-m)^2/2\xi^2]/(2\pi)^{1/2}\xi \tag{1}$$

where m and ξ are energy dependent parameters. It may be helpful to the reader to give a brief resumé of the derivation of equation (1).

Let us assume that, when two hadrons collide, they behave as if made up of a large s-dependent number N of independent scattering centres. At each centre there is activity that may lead to the production of pairs of oppositely charged particles. Thus at centre ν j_ν pairs will be produced following a probability distribution $P_\nu(j_\nu)$ with mean m_ν and variance ξ_ν. Clearly $j = \Sigma_{\nu=1}^N j_\nu$: defining $m = \Sigma_{\nu=1}^N m_\nu$, $\xi^2 = \Sigma_{\nu=1}^N \xi_\nu^2$ and $x = (j - m)/\xi$, we can make use of the central limit theorem of probability which tells us that, subject to certain very general conditions, and neglecting terms of order $N^{-1/2}$, x is normally distributed with unit mean; that is equation (1) holds.

To take account of the discreteness of j, and of the constraint $j \geqslant 0$ we adopt the standpoint of simplicity and replace equation (1) by

$$P(j) = \exp[-(j-m)^2/2\xi^2]/\Sigma \qquad (j \geqslant 0),$$

$$P(j) = 0 \qquad (j < 0)$$

where

$$\Sigma = \sum_{j=0}^{\infty} \exp[-(j-m)^2/2\xi^2]. \tag{2}$$

For the justification of and advantages of this procedure, see the work of Kaiser (1972, 1973). For an alternative procedure, see the work of Parry and Rotelli (1973).

The approximation of equation (1) by equation (2), though of appealing simplicity (as we have said), is nonetheless so naive that we ought to expect to find that it does not fit all of the data on charged multiplicity distributions, especially at low energies. In the original paper on the model by Kaiser (1972), however, which was written at a time when there was virtually no data for P_{LAB} in excess of 30 GeV/c, it was shown that equation (2), with parameters separately determined at each value of P_{LAB}, fitted virtually all of the then available data on charged multiplicity distribution in pp, π^-p and π^+p scattering (for P_{LAB} in excess of 8 GeV/c). That such a simple model could produce such an all-embracing fit was, naturally, extremely pleasing, but it was not surprising that, when data became available from Serpuknov and NAL, equation (1) did not work quite as well.

The discrepancies that arise are shown in figure 1, in which $Q(j) = \ln[\sigma(j)/\sigma(j-1)]$ is plotted as a function of j. From equation (2) we find that

$$Q(j) = (m-j+\tfrac{1}{2})/\xi^2 \tag{3}$$

and the best straight line fits at each value of P_{LAB} are shown. For pp scattering below 30 GeV/c, typified by figures 1(a) and 1(b), the fit is good for all values of j provided that we include σ(el) in $\sigma(j = 0)$. When P_{LAB} exceeds 30 GeV/c, as the example in figure 1(c) shows, the fit breaks down at $j = 1$ irrespective of whether we include σ(el) in or exclude σ(el) from $\sigma(j = 0)$, but is extremely good for other values of j.

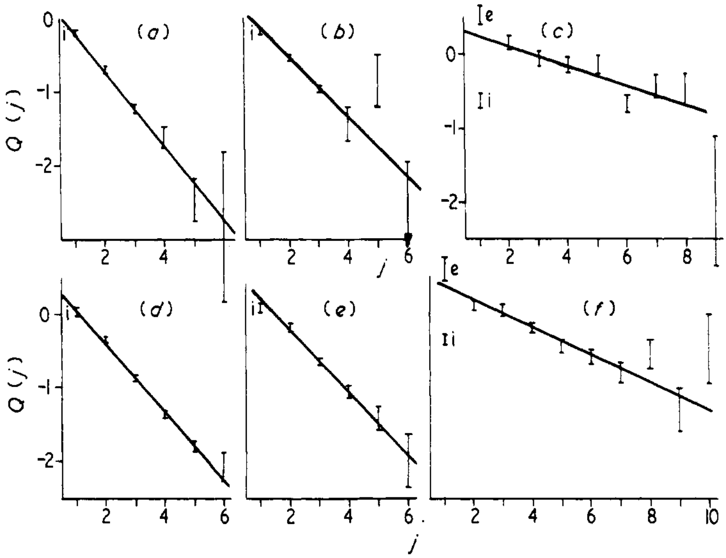

Figure 1. The straight line fits of the approximately normal distribution equation (3) to various data blocks, with parameters determined separately at each value of P_{LAB}. The data are presented in the form of plots of $Q(j) = \ln[\sigma(j)/\sigma(j-1)]$ as a function of j. For $Q(j = 1)$ the points labelled i and e include and exclude σ(el) respectively. pp scattering at: (a) 21 GeV/c, (b) 28·5 GeV/c, (c) 205 GeV/c; π^-p scattering at: (d) 25 GeV/c, (f) 205 GeV/c; K^-p scattering at: (e) 33·8 GeV/c.

This discrepancy was handled by introducing a 'diffractive' mechanism, which contributes negligibly to production amplitudes and is important in the elastic amplitude only. All production cross sections and part of the elastic amplitude are attributed to a non-diffractive mechanism described by equation (2). Again, this is taking the standpoint that it is desirable to look for the simplest possible modifications to the normal distribution (1). The justification for our treatment of the diffractive mechanism is given by Kaiser (1972, 1973). As a result of our assumptions, deviations from equation (2) appear in $P(j = 0)$ so that

$$P(j) = q\delta_{j0} + (1-q)\exp[-(j-m)^2/2\xi^2]/\Sigma \tag{4}$$

where q is an energy dependent parameter. The following parametrizations of q, m and ξ were chosen:

$$\begin{aligned} q &= \alpha + \beta/\ln s \\ m &= a + bs^{c} \\ \xi &= a' + b's^{c'}. \end{aligned} \tag{5}$$

Note that these parametrizations are not unique. The expression for q is motivated by a study of possible Regge exchanges in the elastic amplitude. The parametrizations for m and ξ are designed to give power-law s-dependences of the mean and dispersion (although other forms are possible). The number of parameters can be reduced by one since, as is explained by Kaiser (1973), the independent centre model predicts that $c' = c/2$—but we assert that the details of these parametrizations are unimportant. The resulting best fit to $P(j)$ for pp scattering with P_{LAB} in the range 5–300 GeV/c is shown in figure 2 and is extremely satisfactory—the χ^2 per point is almost exactly one.

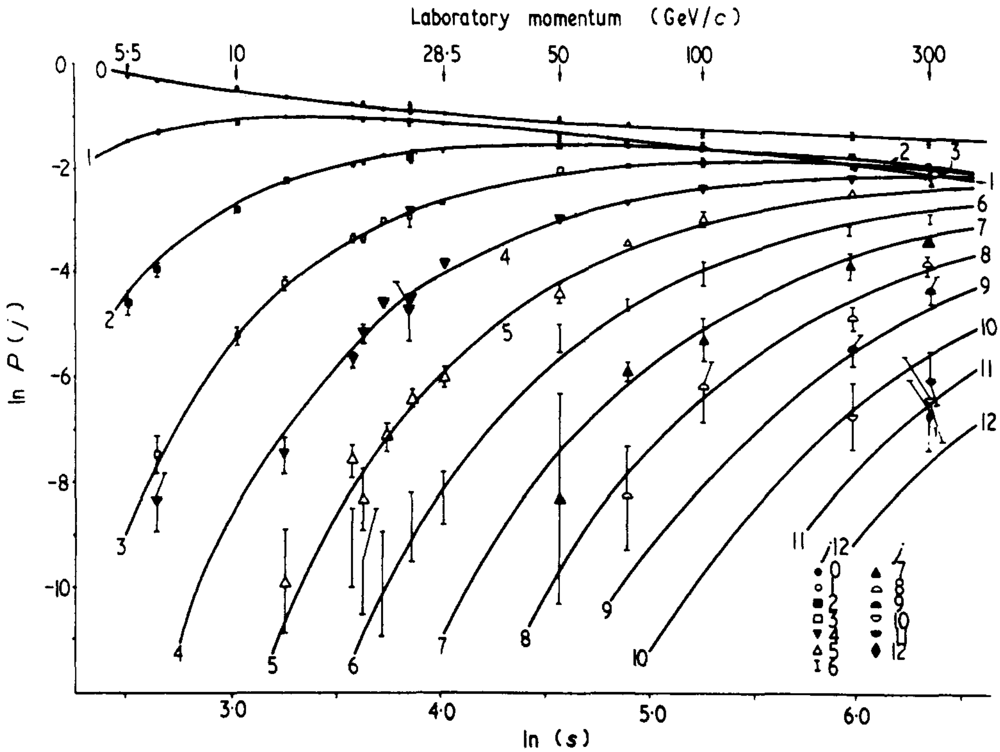

Figure 2. A plot of $P(j)$ against $\ln s$ for pp scattering with values of P_{LAB} in the range 5·5 to 300 GeV/c. $P(j = 0)$ includes σ(el). The full curves are the best fits using equations (4) and (5) and are labelled by the value of j.

3. Extension to π^-p scattering

It is natural to ask whether such a model can work for multiplicity data in other scattering processes. The model (2), with parameters fitted separately at each energy, works as well for $\pi^{\pm}$p and $K^{\pm}$p scattering as it does for pp scattering, as is shown by the examples in figure 1(*d*)–(*f*). All other data blocks are well fitted. The only process for which sufficient data exists to attempt a more ambitious fit as in equations (4) and (5) is π^-p scattering.

Note that we exclude charge exchange scattering from our discussion. We shall also not bother to try and parametrize q, but shall fit only to $P(j \geqslant 1)$—clearly if this procedure is successful, any disagreement with the data for $P(j = 0)$ can be trivially accommodated by choosing a value of q for each value of P_{LAB}. It will then be a simple matter to find a smooth s-dependent parametrization for q and, provided that the result does not differ greatly from that in pp scattering, the form of q can be motivated as for the pp case.

In order to allow for the fact that, by excluding $P(j = 0)$ from our fit, the probabilities no longer sum to one, we remove the normalization by fitting to $Q(j)$, see equation (3), rather than to $P(j)$. The best fit to the $Q(j)$'s appears in figure 3. The χ^2 per point here is approximately two—less satisfactory than in the case of pp scattering. It is conceivable that the neglect of charge exchange processes has some subtle effect on all of the $P(j)$'s. On the other hand, there seems to be some disagreement among the data. For example, the 40 GeV/c data (BBDHSTTUBW collaboration 1973), when fitted by equation (3) at the one value of P_{LAB}, is if anything slightly *broader* than the 50 GeV/c (France–USSR/CERN–USSR collaboration 1973) data. This contradicts the trend of the experimental results and naturally leads to a worsening of the value of χ^2 when an energy dependent parametrization is attempted.

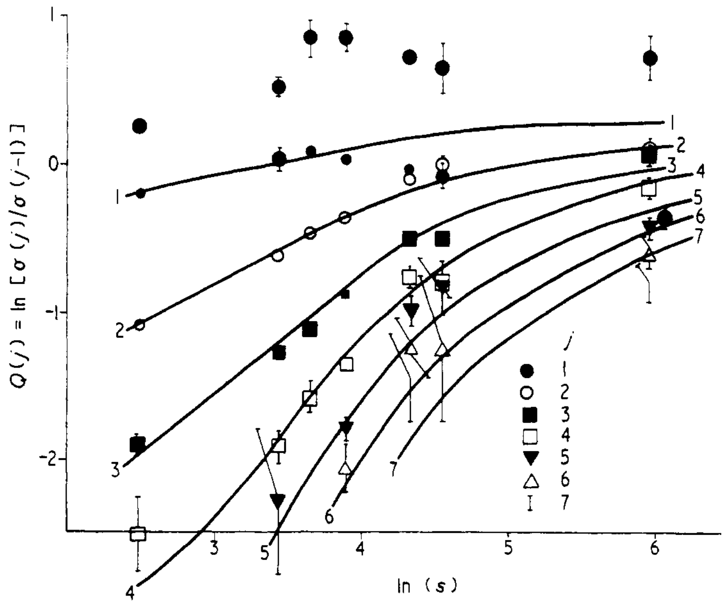

Figure 3. A plot of $Q(j)$ against ln(s) for π^-p scattering with P_{LAB} in the range 10 to 200 GeV/c. For $Q(j = 1)$ the upper and lower points exclude and include σ(el) respectively. The full curves are the best fits using equations (3) and (5) (excluding $Q(j = 1)$ as explained in the text). The curve labelled 1 is the extrapolation of the fit to $j = 1$.

By extrapolating our fit to $j = 1$ (see the curve labelled 1 on figure 3), it is clear that the values of $Q(j = 1)$ predicted by the approximately normal model *without* modification by including a diffractive mechanism are: (*a*) consistent with the experimental values of $Q(j = 1)$ *including* σ(el) for $P_{LAB} < 30$ GeV/c; and (*b*) lie between the $Q(j = 1)$ including or excluding σ(el) as P_{LAB} increases. This is qualitatively similar to what happens in pp scattering. It shows that, in order to change the $j = 1$ curve in figure 3 until it agrees with the lower of the two sets of experimental points for $Q(j = 1)$, it is necessary to modify the elastic amplitude only. It also shows that a parametrization for q similar to that used for the pp data would work reasonably well and justifies our neglect of this trivial task.

4. Comparison of the fits to the π^-p and pp data

In table 1 we present a comparison of the fits to the π^-p and pp data. We see that the parameters are comparable except for a and, to some extent a'. We interpret our results as follows.

(i) The charged particle multiplicity distributions in both π^-p and pp scattering (with P_{LAB} taking on values in the range 5–300 GeV/c) are both well described by the approximately normal distribution, equations (4) and (5), with an additional diffractive component in the elastic amplitude only.

(ii) The diffractive component contributes negligibly to the production amplitudes in both cases.

(iii) As s becomes large, the data are consistent with the hypothesis that the nature of the particles in the initial state becomes irrelevant and that the important parameter in determining the probability distribution is s (or equivalently, at such high energies, E_{av}).

Table 1. The parameters and the values of χ^2 per point for the fits to the pp and π^-p data as described in the text.

Initial state	a	b	c	a'	b'	c'	χ^2/point
pp	−1·95 ±0·01	0·830 ±0·003	0·260 ±0·001	−2·440 ±0·004	2·306 ±0·003	0·133 ±0·001	0·98
π^-p	−0·39 ±0·01	0·78 ±0·01	0·257 ±0·002	−2·33 ±0·01	2·30 ±0·01	0·129 ±0·001	1·96

5. An alternative approach to the data

In the previous paragraph, we have come to an unconventional conclusion, namely that the data are consistent with there being no need to talk of separate diffractive and non-diffractive mechanisms contributing to production amplitudes. Examples of an opposing point of view are (Harari and Rabinovici 1973, Fialkowski and Miettinen 1973, Van Hove 1973, Quigg and Jackson 1972, Frazer *et al* 1972) two component models having the following general features. There is 6–8 mb of diffractive cross section. For pp scattering with P_{LAB} in the range 50–300 GeV/c the bulk of this cross section is split between $\sigma(0)$ and $\sigma(1)$, giving constant contributions to each (and assuming no interference with the non-diffractive mechanism).

Another example, this time of experimental results which are interpreted as evidence for the existence of a distinct diffractive mechanism, is to be found in the work of the CHLM group at the ISR, in which the proton inclusive spectrum is measured at $s = 930\,\text{GeV}^2$ (Albrow *et al* 1973). The spectrum of the proton exhibits a sharp peak when x, the Feynman scaling variable, is close to one. Diffractive processes are defined by making cuts on the proton longitudinal momentum ($x > 0{\cdot}85$) and on the missing mass M ($M^2 \leqslant 50\,\text{GeV}^2$). Such events are supposed to correspond to the excitation and subsequent decay of one incident proton while the other merely suffers a change in momentum. The cross section here is again of the order of 6 mb and agrees with the result of an analysis of the importance of diffractive processes in pp scattering at lower values of P_{LAB} (Pirila and Ruuskanen 1972). Studies of the importance of diffractive excitation in π^-p scattering also exist, see, for example Pokorski and Van Hove (1973).

Confronted with this and other work, the supporter of a model such as the one described in the previous section, or of other models in which only one component is used (for example, models satisfying KNO scaling (Koba *et al* 1972, Slattery 1972)), or the Feynman gas model of order 2 (Frazer *et al* 1973) is forced into one of two positions. He can argue that the model of equations (3) and (4), based on the existence of independent centres of scattering, does not make any statement at all about the momentum distribution of the final state particles. Presumably, after the initial creation of hadrons, there will be complicated final state interactions which determine the momenta and account, for example, for the peak in the proton inclusive spectrum mentioned above. To this argument the objection may be raised that diffractive processes have properties that differ from those of the non-diffractive kind. For example, in the papers on two component models the diffractive component leads to constant cross sections and a constant mean. The non-diffractive component is such that the mean increases and the cross sections of fixed multiplicity vanish as s becomes large. The latest experimental results (Albrow *et al* 1973), however, show that the missing mass spectrum of protons in diffractive processes probably changes as energy increases, suggesting that the mean number of particles in such collisions is a growing function of s and that cross sections of fixed multiplicity may vanish as P_{LAB} becomes very large. Of course, it is possible that the rate of growth of, for example, the mean is still very different from that in non-diffractive processes: on the other hand, the data are already forcing us to blur the distinction between the two components. A more detailed experimental analysis of the behaviour of the two sorts of process as a function of s may yet reveal that there is, in fact, no difference. If there is, why does the simple model of equations (3) and (4) work so well?

The previous discussion represents the adoption of an extreme position. The alternative is to argue that equations (3) and (4) are so simple that, in the complicated world of hadron–hadron scattering, we ought to expect disagreement with at least some of the data. The proper way in which to study the question of the existence or not of a diffractive component which contributes significantly to production processes is to try to extend the model to describe momentum distributions (work on this problem is under way at present). Qualitatively it is difficult to think of a way in which, for example, our simple one component model could reproduce the above mentioned peak in the proton inclusive spectrum. A proper study of momentum distributions within the framework of the model may well reveal that the naive truncation of the approximately normal distribution, equation (2), is not adequate to describe cross sections of low multiplicity and may lead to modifications that allow a substantial diffractive component in production processes.

What we are admitting here is that the data on the $\sigma(j)$'s are probably not sensitive enough to settle the question of the importance of a separate diffractive component. That some progress can be made, however, we shall show by referring the reader to figure 4, in which we have plotted $Q(j) = \ln[\sigma(j)/\sigma(j-1)]$ for pp, π^+p, π^-p, K^+p and K^-p scattering, as a function of the square root of the inverse of the available energy. We have used $(E_{av})^{-1/2}$ because (and only because) this gives a scale along which the data points are reasonably spaced.

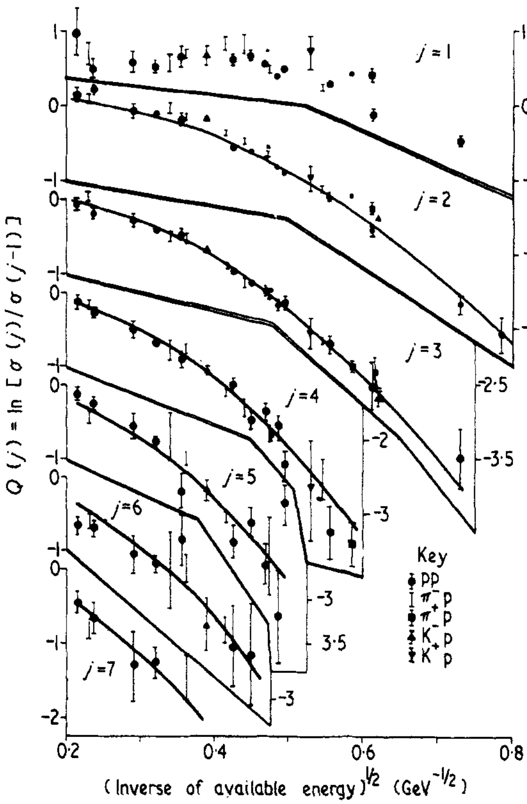

Figure 4. A plot of $Q(j)$ against $(E_{av})^{-1/2}$ for various values of j. The full curves are the best fit of equations (3) and (5) to the data for $j \geq 3$ only. Appearing on the graph are virtually all of the charged particle multiplicity data for pp, $\pi^{\pm}$p and $K^{\pm}$p interactions.

The following conclusion may be drawn from a study of figure 4. Provided that $j > 2$, $Q(j)$ depends only on the available energy and not on the nature of the particles in the initial state. That is, for $j > 2$, our plot consists of truly remarkable universal curves. For $j = 2$, the pp data lies slightly but systematically below the rest (but as E_{av} becomes large, the data may all tend to the same curve). When $j = 1$, the data are not as good as for $j = 2$ but show some dependence on the initial state.

We shall try to interpret these results with a view to making a distinction between those contributions to $\sigma(j)$ that depend strongly on the initial state—diffractive or perhaps 'initial particle' contributions—and those that depend only on E_{av}—non-diffractive or perhaps 'statistical' processes. We therefore assume that $\sigma(j)$ can be split up as follows:

$$\sigma(j) = \sigma_D(j)+\sigma_{ND}(j)+\sigma_{INT}(j) \tag{6}$$

where the subscripts D and ND refer to diffractive and non-diffractive mechanisms respectively and $\sigma_{INT}(j)$ is an interference term.

We note that all of the $\sigma(j)$ are trivially dependent on the 'initial particle' mechanism: the elastic, inelastic and total cross sections σ(el), σ(inel) and σ(tot) are known from experiment to depend on the nature of the colliding hadrons. It is to take account of this overall difference in normalization that we choose to use $Q(j) = \ln[\sigma(j)/\sigma(j-1)]$ instead of $\sigma(j)$ itself:

$$Q(j) = \ln\frac{\sigma_D(j)+\sigma_{ND}(j)+\sigma_{INT}(j)}{\sigma_D(j-1)+\sigma_{ND}(j-1)+\sigma_{INT}(j-1)}. \tag{7}$$

It is clear that, if any pair of $\sigma(j)$ and $\sigma(j-1)$ are dominated by a 'statistical' mechanism (up to the overall normalization) $Q(j)$ will depend only on E_{av}. It follows that a possible interpretation of figure 4 is that all the $Q(j)$ for $j \geqslant 3$ and hence that all the $\sigma(j)$ for $j \geqslant 2$ are dominated by a 'statistical' mechanism.

When $j = 2$, however, the pp data points, as we have seen, lie slightly but systematically below the rest. That is, we have evidence for a dependence on parameters other than E_{av}. We have already written of diffractive processes in terms of the excitation and decay of one (or both) of the hadrons in the initial state. It is conceivable that, at the same value of E_{av}, the decays of, say, an excited pion and an excited proton will differ. The diffractive contribution therefore becomes substantial in $\sigma(1)$ since we have argued that it is small in $\sigma(2)$.

As for $Q(1)$, there seems little evidence that it depends on anything other than E_{av}. On the other hand, the data are not as good as for $j = 1$ and are somewhat erratic. In addition, if both $\sigma(1)$ and $\sigma(0)$ contain substantial contributions from both diffractive and non-diffractive components, there may be subtle cancellations due to the presence of σ_{INT}. The alternative is to assume that the differences in $Q(j = 2)$ mentioned above are accidental, that is, due to fluctuations in the data. In view of the one way nature of the discrepancies, however, this is not likely.

On figure 4 we display our best fit (using equations (3) and (5)) to all the data with $P_{LAB} > 10$ GeV/c and $j \geqslant 3$. The χ^2 per point is about 1·4 with the bulk of the large values of χ^2 coming from points in the π^-p data, as was the case with our previous fit. The fits extrapolate well to lower energies.

If we extrapolate our fit to $j = 2$, the χ^2 per $j = 2$ point for pp scattering is still about one. For all other processes, it is about ten. The data are saying that the deviation in $Q(j = 2)$ from the non-diffractive distribution as parametrized by equation (3) is much smaller for pp scattering than for $\pi^{\pm}$p and $K^{\pm}$p scattering. Superficially, the diffractive mechanism that we have argued to contribute to $\sigma(j = 1)$ is much weaker for pp scattering than for the other initial states. More correctly, if such a diffractive mechanism exists interference effects must be important.

6. Conclusion

(i) We have shown that the approximately normal model, equation (2) with parameters

m and σ determined separately at each value of P_{LAB} is in good agreement with $(Q(j): j \geqslant 2)$ as determined from experimental results in pp, $\pi^{\pm}$p and $K^{\pm}$p scattering with P_{LAB} taking on values in the range 5–300 GeV/c—see the examples in figure 1.

(ii) A more sophisticated energy dependent parametrization using equations (4) and (5) or equations (3) and (5) reproduces essentially all of the pp and π^-p data listed in the references. The only deviation from the approximately normal distribution is accommodated by the introduction of a diffractive component in the elastic amplitude only. This component is parametrized in a way motivated by the study of Regge exchanges. We repeat and expand the assertion made by Kaiser (1972, 1973): this model provides a simple and highly successful description of all of the existing data on multiplicity distributions if P_{LAB} exceeds 5 GeV/c.

(iii) The parametrizations of the pp and π^-p data become the same as s becomes large.

(iv) Motivated by recent studies that suggest that the diffractive mechanism is important in production cross sections, we have tried to study the modifications that are necessary to accommodate such a mechanism. We have concluded that a way of making the model describe momentum distributions must be found (as we have said, this problem is being studied at present). Looking at the plots of data from a variety of initial states, as shown in figure 4, we have concluded that the diffractive mechanism can be substantial in only $\sigma(0)$ and $\sigma(1)$, in agreement with the parametrizations of two component models.

(v) Irrespective of whether or not there is an important diffractive component in production amplitudes, or whether or not equations (3), (4) and (5) are a satisfactory model, figure 4 for $j > 2$ provides a truly remarkable universal fit. Even for $j = 2$ and $j = 1$, the universality is not too bad.

This information can be used to predict, for example, the K^-p multiplicity distribution at 150 GeV/c, a value of the laboratory momentum accessible at NAL. In table 2 we tabulate the predictions using the full curves on figure 4 as interpolations. We expect the predictions to be very good for $j > 2$. For $j = 2$, the experimental result may well be a little higher than the prediction. For $j = 1$, excluding σ(el) from $\sigma(j = 0)$, the experimental result ought also to lie above the prediction. We have not bothered to calculate errors on the prediction using the standard error matrix technique—for given j such errors should certainly be no larger than the rough average of the errors on neighbouring experimental points on figure 4.

Table 2. Predictions of the charged particle multiplicity distribution in K^-p scattering at 150 GeV/c, derived as described in the text.

j	$Q(j)$
1	0·24
2	0·04
3	−0·16
4	−0·35
5	−0·55
6	−0·75
7	−0·95
8	−1·15
9	−1·34
10	−1·54

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